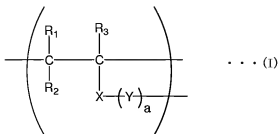
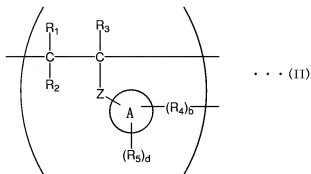


1. A multi-branched polymer having repeating units represented by a formula (I):



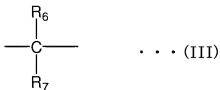
5 wherein R<sub>1</sub> to R<sub>3</sub> each independently represents hydrogen or a hydrocarbon group, R<sub>1</sub> may be bonded to R<sub>3</sub> to form a ring; X represents a connecting group having a valence of 3 or higher; Y may be the same or different and each represents a functional group which may have an active halogen atom; and a is an integer of 2 or larger.

2. The multi-branched polymer according to claim 1, wherein the repeating units  
10 represented by the formula (I) are repeating units represented by a formula (II):



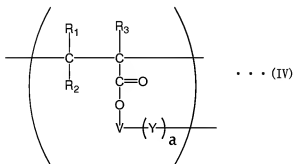
wherein R<sub>1</sub> to R<sub>3</sub> are as defined above; Z represents a single bond or a connecting group having a valence of 2 or higher; A represents an aromatic hydrocarbon group or an aromatic heterocyclic group; R<sub>4</sub> may be the same or different and each represents a functional group which may have an active halogen atom; b is an integer of 2 or larger; R<sub>5</sub> represents a halogen atom or an organic group and d is 0 or an integer of 1 or larger and R<sub>5</sub> may be the same or different when d is 2 or larger.

3. The multi-branched polymer according to claim 2, wherein in the formula (II), Z is a single bond; A is an aromatic hydrocarbon ring; and R<sub>4</sub> is a functional group represented by a formula (III):



5 wherein R<sub>6</sub> and R<sub>7</sub> each independently represents hydrogen, a halogen atom, an alkyl group which may have a substituent, or a linkage with other repeating units with a proviso that R<sub>6</sub> and R<sub>7</sub> do not become linkages with other repeating units at the same time.

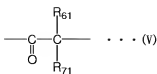
4. The multi-branched polymer according to claim 1, wherein the repeating units represented by the formula (I) are repeating units represented by a formula (IV):



wherein R<sub>1</sub> to R<sub>3</sub>, Y, and a are as defined above; and V represents a connecting group having a valence of 3 or higher.

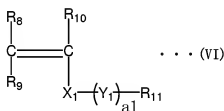
15 5. The multi-branched polymer according to claim 4, wherein V is an alkyleneepoxy group in the formula (IV).

6. The multi-branched polymer according to claim 4 or 5, wherein in the formula (IV), Y is a functional group represented by a formula (V):



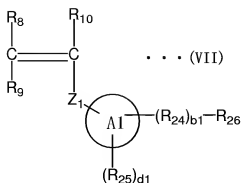
wherein  $\text{R}_{61}$  and  $\text{R}_{71}$  each independently represents hydrogen, a halogen atom, an alkyl group which may have a substituent, or a linkage with other repeating units with a proviso that  $\text{R}_{61}$  and  $\text{R}_{71}$  do not become linkages with other repeating units at the same time.

7. A multi-branched polymer obtained with a living radical polymerization method using a metal catalyst by polymerizing compounds represented by a formula (VI):



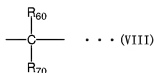
wherein  $\text{R}_8$  to  $\text{R}_{10}$  each independently represents hydrogen or a hydrocarbon group, and  $\text{R}_8$  may be bonded to  $\text{R}_{10}$  to form a ring;  $\text{X}_1$  represents a connecting group having a valence of 3 or higher;  $\text{Y}_1$  may be the same or different and each represents a functional group which may have an active halogen atom;  $a1$  is an integer of 2 or larger; and  $\text{R}_{11}$  represents a chlorine atom, a bromine atom, or an iodine atom.

8. The multi-branched polymer according to claim 7, wherein the compounds represented by the formula (VI) are compounds represented by a formula (VII):



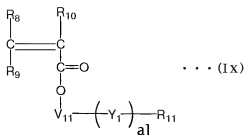
wherein  $\text{R}_8$  to  $\text{R}_{10}$  are as defined above;  $\text{Z}_1$  represents a single bond or a connecting group having a valence of 2 or higher; A1 represents an aromatic hydrocarbon group or an aromatic heterocyclic group;  $\text{R}_{24}$  may be the same or different and each represents a functional group which may have an active halogen atom;  $b1$  is an integer of 2 or larger;  $\text{R}_{25}$  represents a halogen atom or an organic group and  $d1$  is 0 or an integer of 1 or larger and  $\text{R}_{25}$  may be the same or different when  $d1$  is 2 or larger;  $\text{R}_{26}$  represents a chlorine atom, a bromine atom, or an iodine atom.

9. The multi-branched polymer according to claim 8, wherein in the formula (VII),  $\text{Z}_1$  is a single bond, A1 is an aromatic hydrocarbon group, and  $\text{R}_{24}$  is a functional group represented by a formula (VIII):



wherein  $\text{R}_{60}$  and  $\text{R}_{70}$  each independently represents hydrogen, a halogen atom, or a C1 to C6 alkyl group which may have a substituent with a proviso that  $\text{R}_{60}$  and  $\text{R}_{70}$  are not halogen atoms other than fluorine atoms at the same time.

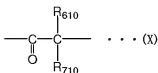
10. The multi-branched polymer according to claim 7, wherein the compounds represented by the formula (VI) are compounds represented by a formula (IX):



wherein R<sub>8</sub> to R<sub>10</sub> are as defined above respectively; V<sub>11</sub> represents a connecting group having a valence of 3 or higher; Y<sub>1</sub> may be the same or different and each represents a functional group which may have an active halogen atom; a<sub>1</sub> is an integer of 2 or larger; and R<sub>11</sub> represents a chlorine atom, a bromine atom, or an iodine atom.

11. The multi-branched polymer according to claim 10, wherein V<sub>11</sub> is an alkylene, polyoxy, or a polyoxoalkylene group in the formula (IX).

12. The multi-branched polymer according to claim 10 or 11, wherein in the formula (IX), Y<sub>1</sub> is a functional group represented by a formula (X):



wherein R<sub>610</sub> and R<sub>710</sub> each independently represents hydrogen, a halogen atom, an alkyl group which may have a substituent, or a linkage with other repeating units with a proviso that R<sub>610</sub> and R<sub>710</sub> do not become linkages with other repeating units at the same time.

13. The multi-branched polymer according to any one of claims claim 1 to 12 or 7, wherein a ratio (Mw/Mn) of weight average molecular weight (Mw) to number average molecular weight (Mn) of the polymer is in a range between 1.01 and 9.99.

14. The multi-branched polymer according to any one of claims claim 1 to 13 or 7, wherein the number average molecular weight (Mn) of the polymer is in a range between

200 and 20,000,000.

15. The multi-branched polymer according to ~~any one of claims claim 1 to 14 or 7~~, wherein the multi-branched polymer is a hyperbranched polymer.

16. A hyperbranched polymer which is branched by a carbon-carbon bond and has a  
5 ratio (Mw/Mn) of weight average molecular weight (Mw) to number average molecular weight (Mn) in a range between 1.01 and 9.99.

17. A hyperbranched polymer obtained by polymerizing a compound having 2 or more polymerization-initiation sites and polymerizable unsaturated bonds by a living radical polymerization method using a metal catalyst.

10 18. The hyperbranched polymer according to claim 16 or 17, wherein the number average molecular weight (Mn) of the polymer is in a range between 200 and 20,000,000.

19. The hyperbranched polymer according to ~~any one of claims claim 16 to 18 or 17~~, wherein the polymer has a functional group at a polymer terminal.

15 20. A star polymer having the multi-branched polymer according to ~~any one of claims claim 1 to 15 or 7~~ or the hyperbranched polymer according to ~~any one of claims claim 16 to 19 or 17~~ as a core thereof.